Enhancing the travel comfort of quarter-car model using fractional order terminal sliding mode controller

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Abstract

The active suspension system used in a car model is beneficial to reduce the vibration of the vehicle and enhance the travel comfort. Different kinds of conventional and intelligent control strategies have been proposed for 2 Degree of Freedom (DOF) quarter car model. In this paper, travel comfort of the passenger is analyzed by designing and simulating the Active Suspension System (ASS), Fractional order Sliding Mode Controller (FSMC) and Fractional order Terminal Sliding Mode Controller (FTSMC) for a quarter car model. These controllers are designed for the perturbed condition of the system dynamics and tested for the normal condition. While testing the performance of the controllers, the system is subjected to three types of road disturbance individually. The responses are compared with each other along with the passive system. The results show that FTSMC reduced the vibration more than the FSMC, TSMC, and passive system. Therefore, the minimum vibration due to the FTSMC enhances the travel comfort.

Keywords: Fractional order SMC; Fractional order Terminal SMC; Ride comfort; Sliding mode Control; Terminal SMC

1. Introduction

The suspension system in a vehicle serves as a useful mean to provide ride comfort and partly contribute to the vehicle handling. Passive, semi-active and active suspensions are the three types of suspension system used in the vehicles. Different kinds of springs are employed in the passive suspension system. In semi-active suspension, the spring and damper parameters are changed with respect to the road condition during the runtime. Hence vibration reduces to a particular level and it requires minimum power to perform. In case of Active Suspension System (ASS), the external control force is applied in the opposite direction of the vibration. Therefore ASS reduces the vibration better than semi-active and passive suspension systems. The active and semi-active suspension systems are used in addition to the passive suspension system in the vehicles.

While travelling in a car, comfort of the passenger will be more when there is a minimum vibration. Since the road surfaces are not smooth, the vehicle and the passengers will experience the vibrations. These vibrations affect the health of the passengers and the lifespan of the vehicle. The wear and tear of the vehicle will also increase due to vibrations. Therefore, an ASS is designed to handle these vibrations. Different kinds of algorithms are used to produce the control force in ASS. The control force should be as minimum as possible and should change rapidly with respect to the changes in the road disturbances.

Considering the one fourth of the vehicle for the analysis of the vibration control is a common practice among the researchers [1,2]. These type of quarter car model has two masses namely sprung mass and unsprung mass [3]. The control strategies for the Quarter car are available in many research papers and the same is reviewed [4]. Optimization of PID controller for an Active suspension system is discussed in [5]. Regarding the Sliding Mode Control (SMC), a semi-active suspension controller using SMC for quarter car with driver model was discussed [6]. Grey Fuzzy SMC [7], type-2 fuzzy logic based SMC [8], Robust Fuzzy based SMC [9], and model-free adaptive SMC [10] have been designed to enhance the performance of the quarter car model. In [11] SMC was used to estimate the car body mass of the quarter car. Fractional order SMC (FSMC) [12-16] and Terminal SMC (TSMC) [17-20] have been used in various applications to improve the performances of the system. Fractional order Terminal SMC (FTSMC) was employed for a dynamical system with uncertainty in Ref. [21]. In this paper, FTSMC is proposed to enhance the travel comfort of a quarter-car model. The performance of the con-
controller is compared with FSMC, TSMC and the passive suspension system. Three types of road profiles are considered to test the performance of the controller. This paper is organized as follows. In Section 2, quarter-car model is discussed. In Section 3, controllers design approaches for the proposed model is presented. In section 4, the numerical simulation and results are discussed. Finally, conclusions are summarized in Section 5.

2. Quarter car model

The vertical vibration of a vehicle using a quarter-car model can be represented by a 2-DOFs system. The sprung and unsprung masses are denoted by \(m_s\) and \(m_u\), respectively. The sprung mass \(m_s\) represents a quarter of the vehicle body, the unsprung mass \(m_u\) represents one wheel of the vehicle. A spring with stiffness \(k_s\) and a shock absorber with viscous damping coefficient \(c_s\) support the sprung mass and these arrangements are called as the passive suspension.

The \(m_u\) is in direct contact with the ground through the spring \(k_s\), representing the tire stiffness. Fig. 1 shows the quarter car model with active suspension system.

\[
m_u\ddot{y}_u = -k_s(y_u - y_r) - c_s(\dot{y}_u - \dot{y}_r) + k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - f_s
\]

\[
m_s\ddot{y}_s = -k_s(y_s - y_u) - c_s(\dot{y}_s - \dot{y}_u) + f_s
\]  

(1)

(2)

3. Design of Controllers

To control the vibration in the vehicle, SMC is one of the decent choices because of the uncertainties in the road disturbance and considering that SMC is of the simple and effective robust control methods. SMC produces the control signal to the sliding surface and converges to zero in finite time situation.

In order to design the SMC, the required Lyapunov’s function is considered as \(V = \frac{1}{2} s^2\). The existence condition for sliding mode is possible when

\[
\dot{V} = ss_1 < 0
\]  

(3)

where \(s\) is the sliding surface.

The state variables \(x_1\) and \(x_2\) are chosen as follows. The suspension deflection is \(x_1 = y_s - y_r\), and the car body velocity is \(x_2 = \dot{y}_s\). The essential condition to drive the state trajectory toward the sliding surface is [24]:

\[
\dot{s}(x, t) = 0
\]  

(4)

3.1. Design of Terminal Sliding Mode Controller

To design TSMC, state variables are chosen as shown in Eq. 2 and 3. The terminal sliding surface \(S\) becomes

\[
S = \frac{p}{q} f_s
\]  

(5)

where \(c\) is a surface gain, \(p\) and \(q\) are integers which satisfy the condition \(p < q < 2p\) [17]. The derivative of Eq. 5

\[
\dot{s} = \frac{p}{q}(\dot{x}_1)\dot{s} + \dot{x}_2 \dot{s} + c\dot{x}_2
\]

(6)

Subsequently

\[
0 = \frac{p}{q}(\dot{x}_1)\dot{s} + \dot{x}_2 \dot{s} + c\dot{x}_2
\]  

(7)

By substituting the state variables above, the \(f_s\) becomes \(f_s_{,equ} = \) \(f_s\) \(\text{sign}(s)

\[
f_{s,\text{equ}} = k_s(y_s - y_u) + c_s(y_s - y_u) - c_q m_s (\dot{y}_s - \dot{y}_u)
\]  

(8)

Hence the desired control force \(f_s\) is,

\[
f_s = f_{s,\text{equ}} - k_s \text{sign}(s)
\]

(9)

\[
f_s = k_s(y_s - y_u) + c_s(y_s - y_u) - c_q m_s (\dot{y}_s - \dot{y}_u)
\]  

(10)

where \(k_m, \text{sign}(s)\) is the switching control and the term \(k = n + f\) which satisfies the desired criteria (Eq. 1) brings the system in to the sliding surface and converges to zero in finite time and \(n\) is a positive constant and

\[
f = -k_s m_s (y_s - y_u) - c_s m_s (y_s - y_u)
\]

(11)

3.1.1. TSMC under perturbed condition

In the QC model, the sprung mass is considered as the constant value. In real time this mass may vary [25] because it includes the mass of the passenger or driver. Therefore, while designing the SMC this variation is also considered and it is termed as perturbed condition. These variations of \(m_s\) should be considered so that the simulation is leading towards the real time situation.

when the sprung mass is varying with respect to time then the \(m_s\) is changed into \(\dot{m}_s\). The hat denotes for the perceived parameters which may be different from the true parameters. Eqs. 8 and 10 can be rearranged as follows
\[ f_{e,\text{eq}} = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - \frac{q}{p} \dot{m}_s(\ddot{y}_s - \ddot{y}_u)^{2-p} \]

Hence the desired control force is,
\[ f = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - \frac{q}{p} \dot{m}_s(\ddot{y}_s - \ddot{y}_u)^{2-p} \]
\[ - k_m \text{sign}(s) \]

where \( k = \beta(\lceil F \rceil + n) + (\beta - 1)\left| f_{s,\text{equ}} \right| \)
\[ \beta = \frac{m_{\text{max}}}{m_{\text{min}}} \]
\[ F = f - \hat{f} \]
\[ \hat{f} = - \frac{k_s}{m_s}(y_s - y_u) - \frac{c_s}{m_s}(\dot{y}_s - \dot{y}_u) \]
\[ f \] is defined in Eq. 11

3.2. Design of Fractional Order Sliding Mode Controller

Fractional calculus is one of the commonly used techniques to extend the integer order calculus into non-integer order calculus. Which means the order can be in the non-integer form [26-28, 13]. The basic operation of the fractional calculus is defined as
\[ aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t-\tau}^{t} f^{(n)}(\tau) (t-\tau)^{\alpha-n+1} d\tau \]

where \( a \) and \( t \) are the upper and lower limits of the fractional operation, \( \alpha \) is the order of the fractional operation. Various definitions for fractional calculus are implemented, but the commonly used definition is Caputo fractional calculus, which is described as:
\[ aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t-\tau}^{t} f^{(n)}(\tau) (t-\tau)^{\alpha-n+1} d\tau \]

where \( m-1 < \alpha < m, m < N \) and from the above definition it is clear that fractional calculus has higher degrees of freedom than integer order calculus. To get the better control performance appropriate fractional order values are assigned for FSMC.

The fractional sliding surface is,
\[ s = D^\alpha x_1 + cx_1 \]

The derivative of the Equation 3.20 is
\[ \dot{s} = D^{\alpha-1} \dot{x}_1 + cx_1 \]

And one may write Eq. 21 as:
\[ 0 = D^{\alpha-1} \dot{x}_1 + cx_1 \]

By substituting the state variables and simplification the equivalent control force is,
\[ f_{s,\text{equ}} = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - D^{1-\alpha} \dot{m}_s(\ddot{y}_s - \ddot{y}_u) \]
\[ - k_m \text{sign}(s) \]

Hence the desired control force is,
\[ f = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - D^{1-\alpha} \dot{m}_s(\ddot{y}_s - \ddot{y}_u) \]
\[ - k_m \text{sign}(s) \]

3.2.1. FSMC under perturbed condition

Under perturbed condition of the Eqs. 23 and 24 are changed as follows
\[ f_{s,\text{equ}} = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - D^{1-\alpha} \dot{m}_s(\ddot{y}_s - \ddot{y}_u) \]

Hence the desired control force is,
\[ f = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - D^{1-\alpha} \dot{m}_s(\ddot{y}_s - \ddot{y}_u) \]
\[ - k_m \text{sign}(s) \]

3.3. Design of Fractional order Terminal Sliding Mode Controller

To design FTSMC, fractional order terminal sliding surface is chosen as follows,
\[ S = (D^\alpha x_1)^{\frac{p}{q}} + cx_1 \]

where \( c \) is the sliding surface gain and \( D^\alpha(\cdot) \) is the fractional calculus with \( 0 < \alpha < 1 \)

The derivative of Eq. 27 is
\[ \dot{S} = \frac{p}{q} (D^\alpha x_1)^{\frac{p-1}{q}} D^{\alpha-1} \dot{x}_1 + cx_1 \]

That can be described as:
\[ 0 = \frac{p}{q} (D^\alpha x_1)^{\frac{p-1}{q}} D^{\alpha-1} \dot{x}_1 + cx_1 \]

By substituting the state variables and simplification the equivalent control force is,
\[ f_{s,\text{equ}} = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - \frac{q}{p} \dot{m}_s(D^{1-\alpha})^\frac{p}{q}(\ddot{y}_s - \ddot{y}_u)^{2-p} \]

Hence the desired control force is,
\[ f = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - \frac{q}{p} \dot{m}_s(D^{1-\alpha})^\frac{p}{q}(\ddot{y}_s - \ddot{y}_u)^{2-p} \]
\[ - k_m \text{sign}(s) \]

3.3.1. FTSMC under perturbed condition

Under the perturbed condition, Eqs. 30 and 31 are changed
as following. The equivalent control force get from above equation is,

\[ f_{\text{eq}} = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) \]

\[ -\frac{q}{p} \hat{m}_s (D^\alpha)^{\frac{p}{q}} (\ddot{y}_s - \ddot{y}_u)^{\frac{p}{q}} \]

(32)

Hence the desired control force is,

\[ f_s = k_s(y_s - y_u) + c_s(\dot{y}_s - \dot{y}_u) - \frac{q}{p} \hat{m}_s (D^\alpha)^{\frac{p}{q}} (\ddot{y}_s - \ddot{y}_u)^{\frac{p}{q}} \]

\[ -k m \text{sign}(s) \]

(33)

4. Numerical Simulations and results

The quarter-car system and the designed sliding mode controllers are simulated in SIMULINK blocks of MATLAB R2012b and the required functions for the road profiles are written as .m files and used in interpreted functional block of the Simulink. The vehicle parameters used in this work are from [29] and summarized in the Table 1 and the data used for the SMC are tabulated in Table 2. All the three controllers are designed with the single bump as road input and then tested with other two types of road input without any modification in the controllers. The first type of road input to the system is single bump [30], the second type of road profile chosen for analysis is the sinusoidal road [31] and third type is a random road profile [32] are shown in Fig. 2. The simulations are performed for the period of 2 seconds in all cases and the responses are plotted. Fig. 3 shows that the perturbed condition for the mass which are varying as 290 + 60 sin (t) [25], i.e., from 230 to 350 kg, where t is time in seconds.

The sprung mass acceleration is the final control element considered for analysis. It is also named as the Body Acceleration (BA). Initially the system is subjected to the single bump road disturbance. The responses are plotted for the designed controllers. Fig. 4 shows the time response of the controllers with single bump input under normal condition and its corresponding forces are shown in Fig. 5. Similarly Fig. 6 shows the response and Fig. 7 shows the forces under the perturbed condition. This force is not settled to zero due to the presences of change in sprung mass continues with time t.

Table 1. Quarter car Parameters

<table>
<thead>
<tr>
<th>Mass (Kg)</th>
<th>Damping coefficient (Ns/m)</th>
<th>Spring stiffness (N/m) X 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_s</td>
<td>59</td>
<td>0</td>
</tr>
<tr>
<td>m_u</td>
<td>290</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 2. SMC Parameters

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Name</th>
<th>Constant Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>n</td>
<td>0.00001</td>
</tr>
<tr>
<td>2</td>
<td>c</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>p</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>q</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>α</td>
<td>0.85</td>
</tr>
<tr>
<td>6</td>
<td>β</td>
<td>1.2336</td>
</tr>
<tr>
<td>7</td>
<td>m_{\text{mass}}</td>
<td>350 kg</td>
</tr>
<tr>
<td>8</td>
<td>m_{\text{min}}</td>
<td>230 kg</td>
</tr>
</tbody>
</table>

The sprung mass acceleration is the final control element considered for analysis. It is also named as the Body Acceleration (BA). Initially the system is subjected to the single bump road disturbance. The responses are plotted for the designed controllers. Fig. 4 shows the time response of the controllers with single bump input under normal condition and its corresponding forces are shown in Fig. 5. Similarly Fig. 6 shows the response and Fig. 7 shows the forces under the perturbed condition. This force is not settled to zero due to the presences of change in sprung mass continues with time t.
Figure 5. Force produced by controllers for single bump input normal condition.

Figure 6. Time response of the BA for single bump input perturbed condition.

Figure 7. Force produced by controllers for single bump input perturbed condition.

Figure 8. Time response of the BA for sinusoidal road input normal condition.

Figure 9. Force produced by controllers for sinusoidal road input normal condition.

Figure 10. Time response of the BA for sinusoidal road input perturbed condition.
The controller’s performances are analysed with respect to the continuous and periodic road disturbances and therefore the sinusoidal road input is considered. Fig. 8 shows the time responses of the controllers with sinusoidal input under normal condition wherein the corresponding forces are shown in Fig. 9. Similarly the Fig. 10 shows the response and Fig. 11 shows the forces under the perturbed condition. Though the magnitudes of the road inputs are same, the periodic disturbance due to sinusoidal input increases the magnitude of the passive response therefore the force required for sinusoidal road input is relative high with respect to the single bump.

The road disturbances are unpredictable therefore it is necessary to consider the random road disturbance to the system. Fig. 12 shows the time response of the controllers with random road input under normal condition and its corresponding forces are shown in Fig. 13. Similarly Fig. 14 shows the response and Fig. 15 shows the forces under the perturbed condition.

The performance of the controllers are analysed in terms of the Root Mean Square (RMS) values of the responses. The RMS values computed and tabulated in Table 3 for the normal condition and Table 4 for the perturbed condition. From these
values, the % of reduction in the BA by the controllers is calculated with respect to passive response for the three types of road profiles.

**Table 3. RMS values of Body acceleration in normal condition (m/s²)**

<table>
<thead>
<tr>
<th>Normal Condition</th>
<th>Types of Road inputs</th>
<th>Single bump</th>
<th>Random</th>
<th>Sinusoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>2.233</td>
<td>2.234</td>
<td>7.411</td>
<td></td>
</tr>
<tr>
<td>FSMC</td>
<td>0.293</td>
<td>0.55</td>
<td>0.9754</td>
<td></td>
</tr>
<tr>
<td>TSMC</td>
<td>0.4079</td>
<td>0.6752</td>
<td>1.452</td>
<td></td>
</tr>
<tr>
<td>FTSMC</td>
<td><strong>0.2803</strong></td>
<td><strong>0.5238</strong></td>
<td><strong>0.9314</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4. RMS values of Body acceleration in perturbed condition (m/s²)**

<table>
<thead>
<tr>
<th>Perturbed Condition</th>
<th>Types of Road inputs</th>
<th>Single bump</th>
<th>Random</th>
<th>Sinusoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>2.1690</td>
<td>1.9100</td>
<td>6.409</td>
<td></td>
</tr>
<tr>
<td>FSMC</td>
<td>0.4072</td>
<td>0.5337</td>
<td>1.376</td>
<td></td>
</tr>
<tr>
<td>TSMC</td>
<td>0.6591</td>
<td>0.8151</td>
<td>2.171</td>
<td></td>
</tr>
<tr>
<td>FTSMC</td>
<td><strong>0.3869</strong></td>
<td><strong>0.5270</strong></td>
<td><strong>1.257</strong></td>
<td></td>
</tr>
</tbody>
</table>

Under normal condition, the FTSMC reduces the BA by 84.14 % which is better than the TSMC (77.45%) and FSMC (83.36%). When the system is subjected to the perturbed condition, the FTSMC reduces the BA by 78.67 % which is better than the TSMC (64.40%) and FSMC (78.15%). In both case the FTSMC controls the BA better than TSMC and FSMC.

With respect to the time response of BA, the performance of these controller under perturbed condition is relatively less than the normal condition because the control force produce by the controllers are also less than that of the controllers under normal condition. Since the sprung mass is varying with respect to time, the control force is generated by considering this variation and hence the performances of the perturbed condition is less than the performances under normal condition. The control force produce by the FTSMC is almost same as the control forces produced by the TSMC and FSMC at the same time the performance of the FTSMC is better than the FSMC and TSMC for normal as well as perturbed condition.

The ride quality is also analyzed with respect to Spectrum Density (PSD) and plotted (shown in Fig. 16 to Fig. 21) for the BA as a function of frequency for all three types of road profiles. FTSMC has reduced BA effectively in the human sensitive frequency range of 4 to 8 for all the road inputs with normal and perturbed conditions [2].

![Figure 16. PSD for single bump input Normal condition](image1)

![Figure 17. PSD for sinusoidal road input Normal condition](image2)

![Figure 18. PSD for Random road input normal condition](image3)

![Figure 19. PSD for single bump input perturbed condition](image4)
5. Conclusions

The vibration control strategies for the quarter car system with FTSMC are designed and simulated to enhance the travel comfort. The performances of the FTSMC are compared against the FSMC, TSMC and passive system. The results are analysed in terms of RMS and PSD. The performance of the FTSMC is 6.69 % better than TSMC and 0.78 % better than FSMC for the normal condition. In case of perturbed condition FTSMC is 14.27 % better than TSMC and 0.52 % better than FSMC. The reduction in vibration is good for all the three types of road profiles. As for as the quarter car model is concerned the sprung mass may be considered as the individual masses such as mass of the frame, seat and driver body. This system can be analysed with these controllers so that more realistic quarter car model is considered for analysis. Other types of advanced sliding mode controllers such as Fraction order Fuzzy SMC can be simulated to enhance the travel comfort.

References


