Rubber hyperelastic constitutive model and its application in finite element analysis of radial tire

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Abstract

The parameters of five typical rubber hyperelastic constitutive models, including Mooney-Rivlin, Ogden, Neo-Hooke, Yeoh and Arruda-Boyce, are obtained in ABAQUS by using parameter fitting methods which are based on three types of combination of different test data, namely uniaxial tension; uniaxial tension and biaxial tension; uniaxial tension, biaxial tension and planar tension. Rubber stress-elongation equations are derived from different strain energy potential functions. Stress-strain curves of uniaxial tension, biaxial tension and planar tension are generated based on the aforementioned stress-elongation equations by adopting the parameters from each fitting method in ABAQUS. These curves are then compared with experimental stress-strain curves to analyze their goodness of fit, where coefficient of determination $R^2$ is used to judge the optimal parameter fitting method. With the optimal parameter fitting method, the goodness of fit for different strain ranges of the above five models are analyzed. In order to validate the above methodology, a case study based on Yeoh constitutive model is carried out. A radial tire finite element model is used to simulate tire stiffness considering Yeoh model parameters generated from three different parameter fitting methods. It is demonstrated by comparing experimental and simulation stiffness results that Yeoh model with parameters fitted from the aforementioned optimal method can give the best accuracy of finite element tire model.

Keywords: Hyperelastic; Constitutive model; Finite element method; Material test; Radial tire

1. Introduction

Rubber constitutive models have been developed for many years in order to precisely represent rubber nonlinear behavior. The early constitutive model is in polynomial form or Ogden form based on continuum mechanics vs. Mooney-Rivlin model [1-2], Ogden model [3,5] and Yeoh model [4]. Then models based on the thermo- dynamics statistics theory appear, which mainly includes Neo-Hooke model [5], Kuhn-Grun model [6], Arruda-Boyce [6] and Gent model [7].

Rubber material constitutive models have been applied widely in the study of rubber components, such as tire. Yintao et al. [8] discussed the rubber material constitutive model and its application in finite element analysis under large deformation. Yanfeng et al. [9] and Xiaofang et al. [10] summarized the statistical thermodynamic model and the continuum mechanics model for hyperelastic materials, respectively. Liangsen et al. [11] present a summary of functions for incompressible isotropic hyperelastic material. Yu Jianhua et al. [12] also discussed the influence of incompressibility on the stress-strain relation of hyperelastic material. Tschoegl [2] studied the reason that Mooney-Rivlin model is unable to predict multi-axial data is lack of expansion of the model and the model needs higher order expansion.

The parameter acquisition of hyperelastic constitutive models was also investigated by many researches [13-17]. Youjian et al. [18] summarized the experimental methods of determining hyperelastic constitutive models. Dengxiang [19] provided mathematical derivation and discussion on experimental design of uniaxial tension test, uniaxial compression test, planar tension test and biaxial tension test.

In tire industry, the application of hyperelastic constitutive model is mainly in the finite element analysis of tire performance. Zhiquo et al. [20] discussed the finite element simulation for the transient dynamic response of rubber elastic wheels. The modeling method based on mechanical constitutive relation of rubber materials has also been studied. Xiaoxiang et al. [21] established an axial symmetric nonlinear finite element tire model, which takes into account the nonlinearity and incompressibility of rubber material, to simulate rim positioning process, inflation and free rotation of tire.

Although there are many achievements in the development of hyperelastic constitutive models and their applications, there is still a lack of analyzing the comprehensive parameter fitting methods and model application accuracy. Therefore, this paper analyzes and contrasts different parameter fitting methods by using uniaxial, biaxial and planar tensile test based on five common hyperplastic constitutive models. A concept of goodness of fit is used to judge the performance of different fitting methods. The optimal parameter fitting method is figured out by evaluation, and then applied to constituu 
tive model for the accuracy analysis in different strain ranges. Finally, a case study based on finite element tire model is adopted to verify the conclusion from the aforementioned study. It is proved that the rubber constitutive model with parameters generated from the optimal fitting method can ensure a high simulation precision in finite element analysis.

2. Experimental Phase

Uniaxial tension test, biaxial tension test and planar tension test were carried out with tire rubber under room temperature (25°C). The uniaxial tension and the planar tension test were completed at 80% elongation, and the biaxial tension test was completed at 60% elongation. The engineering stress-strain curves were obtained by test reflects the mechanical behavior of rubber material as shown in Fig. 1. The stress-strain curves are typical "S" shape curve. The "S" shape curve mainly has three stages: approximate elastic stage, softening stage and hardening stage.

3. Material Model and Parameter Fitting

For the five common hyperelastic material constitutive models, namely Mooney-Rivlin model, Ogden model, Neo-Hooke model, Yeoh model and Arruda-Boyce model, the model parameters are fitted by three different fitting methods. The precision of constitutive model using parameters obtained from the three different fitting ways are analyzed. At last, the optimal fitting method is determined.

3.1 Engineering Stress-Elongation Equation

Mooney-Rivlin, Ogden, Neo-Hooke, Yeoh and Arruda-Boyce model are the five common constitutive models of hyperelastic material. The strain energy potential (SEP) functions U of these models are listed below.

\[
U = \sum_{i=1}^{3} C_i (I_i - 3) + \frac{1}{D_1} (J^a - 1)^2
\]

\[
U = \sum_{i=1}^{3} \frac{2\mu}{a_i^2} (\lambda_i^n + \lambda_i^m + \lambda_i^{2n} - 3) + \frac{1}{D_1} (J^a - 1)^2
\]

\[
U = C_{10} (I_1 - 3) + \frac{1}{D_1} (J^a - 1)^2
\]

\[
U = \sum_{i=1}^{3} C_{i0} (I_i - 3) + \frac{1}{D_1} (J^a - 1)^2
\]

\[
U = \mu \left[ \frac{1}{2} (I_1 - 3) + \frac{1}{20a_i^2} (F_i^2 - 9) + \frac{11}{105a_i^4} (F_i^3 - 27) \right]
\]

\[
U = \mu \left[ \frac{19}{7000a_i^6} (F_i^5 - 81) + \frac{519}{63750a_i^8} (F_i^7 - 243) \right] + \frac{1}{D_1} \left( J^a_1 - 1 \right)^2
\]

For the uniaxial tension, the elongation in the loading direction (direction 1) is the main elongation \( \lambda \), namely:

\[
\lambda_1 = \lambda
\]

The rubber is considered as an incompressible material, then:

\[
J = \lambda_1 \lambda_2 \lambda_3 = 1
\]

Thus

\[
\lambda_2^2 = \lambda_3^2 = \lambda^{-1}
\]

Then, the relationship between the engineering stress \( \sigma_e \) and the strain energy potential for uniaxial tension can be obtained by partial derivative of the strain energy potential U on elongation, as following:

\[
\sigma_e = \frac{\partial U}{\partial \lambda} = 2(1 - \lambda^{-1}) \left( \frac{\partial U}{\partial I_1} + \frac{\partial U}{\partial I_2} \right)
\]

For the biaxial tension, the elongation of the loading direction of the sample is direction 1 and 2, and the elongation in the two directions is consistent. It can be expressed as

\[
\lambda_1 = \lambda_2 = \lambda
\]

The rubber is considered as an incompressible material, then

\[
J = \lambda_1 \lambda_2 \lambda_3 = 1
\]

Thus

\[
\lambda_3 = \lambda^{-2}
\]

Then, the relationship between the engineering stress \( \sigma_e \) and the SEP for biaxial tension can be obtained by partial derivative of the SEP function U on elongation, as following:

\[
\sigma_e = \frac{\partial U}{\partial \lambda} = 2(\lambda - \lambda^{-2}) \left( \frac{\partial U}{\partial I_1} + \frac{\partial U}{\partial I_2} \right)
\]

For the planar tension, the elongation of the loading direction of the sample is direction 1. It can be expressed as

\[
\lambda_1 = \lambda
\]

\[
\lambda_2 = 1
\]

The rubber is considered as an incompressible material, then

\[
J = \lambda_1 \lambda_2 \lambda_3 = 1
\]

Thus

\[
\lambda_3 = \lambda^{-1}
\]

Then, the relationship between the engineering stress \( \sigma_e \) and the SEP for planar tension can be obtained by partial derivative of the SEP function U on elongation, as following:

\[
\sigma_e = \frac{\partial U}{\partial \lambda} = 2(\lambda - \lambda^{-1}) \left( \frac{\partial U}{\partial I_1} + \frac{\partial U}{\partial I_2} \right)
\]

For all the case, the elongation \( \lambda_i \), \( I_1 \) and \( I_2 \) is expressed as

\[
\lambda = 1 + \varepsilon_e
\]

\[
I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2
\]

\[
I_2 = (\lambda_1 \lambda_2)^2 + (\lambda_2 \lambda_3)^2 + (\lambda_3 \lambda_1)^2
\]

Here, \( \varepsilon_e \) is the engineering strain, \( \lambda_i \) is the elongation of the direction 1, \( \lambda_2 \) is the elongation of the direction 2, and \( \lambda_3 \) is the elongation of the direction 3.

Finally, according to the SEP function \( U(I_1, I_2) \) and the re-
The engineering stress-strain relationship \( \sigma = \sigma_E(\lambda, U, I_1, I_2) \) between the engineering stress and strain energy potential function of the material model, the stress-elongation relationship \( \sigma = \sigma_E(\lambda) \) of different tensile types for each material model is derived.

From Eqs. (1) to (19), the engineering stress-elongation equation \( \sigma = \sigma_E(\lambda) \) of the five common models, viz. Mooney-Rivlin model, Ogden model, Neo Hooke model, Yeoh model and Arruda-Boyce model are further derived for the three different tensile types, namely uniaxial tension, biaxial tension and planar tension, as shown in Table 2.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full name</th>
<th>Abbreviation</th>
<th>Full name</th>
</tr>
</thead>
<tbody>
<tr>
<td>UT</td>
<td>Uniaxial tension</td>
<td>Test UT</td>
<td>Test stress-strain curve of UT</td>
</tr>
<tr>
<td>BT</td>
<td>Biaxial tension</td>
<td>Test_BT</td>
<td>Test stress-strain curve of BT</td>
</tr>
<tr>
<td>PT</td>
<td>Planar tension</td>
<td>Test_PT</td>
<td>Test stress-strain curve of PT</td>
</tr>
</tbody>
</table>

Table 1. Abbreviation of the full name

![Engineering stress-strain curves of different tensile tests](image)

Table 2. The parameters list of the material model obtained by the three fitting methods in ABAQUS

<table>
<thead>
<tr>
<th>Group</th>
<th>Neo-Hooke</th>
<th>Mooney-Rivlin</th>
<th>Ogden</th>
<th>Yeoh</th>
<th>Arruda-Boyce</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( C_{m1} )</td>
<td>( C_{m2} )</td>
<td>( C_{m3} )</td>
<td>( C_{m11} )</td>
<td>( C_{m20} )</td>
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<tr>
<td>Para. A</td>
<td>0.6688</td>
<td>0.7253</td>
<td>-0.0924</td>
<td>0.8505</td>
<td>-0.4531</td>
</tr>
<tr>
<td>Para. B</td>
<td>0.6307</td>
<td>0.6409</td>
<td>-0.0035</td>
<td>0.7246</td>
<td>-0.1786</td>
</tr>
<tr>
<td>Para. C</td>
<td>0.6286</td>
<td>0.6349</td>
<td>-0.0013</td>
<td>0.7132</td>
<td>-0.1719</td>
</tr>
</tbody>
</table>

Figure 1. Engineering stress-strain curves of different tensile tests

Table 3. Abbreviation of the full name

Para._A The group of parameter obtained by fitting method A A_Caculate_UT/PT The stress-strain curve of UT/PT calculated with Para._A by equation in Table 2
Para._B The group of parameter obtained by fitting method B B_Caculate_UT/PT The stress-strain curve of UT/PT calculated with Para._B by equation in Table 2
Para._C The group of parameter obtained by fitting method C C_Caculate_UT/PT The stress-strain curve of UT/PT calculated with Para._C by equation in Table 2

Table 4. The parameters list of the material model obtained by the three fitting methods in ABAQUS
Table 2. Engineering stress-elongation equation of three tensile state

<table>
<thead>
<tr>
<th></th>
<th>Uniaxial</th>
<th>Biaxial</th>
<th>Planar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mooney-Rivlin</td>
<td>( \sigma_E = 2\left(\lambda - \lambda^{-3}\right)\left(\lambda^{-3}C_{10} + C_{01}\right) )</td>
<td>( \sigma_E = 2\left(\lambda - \lambda^{-3}\right)\left(C_{10} + \lambda^{3}C_{01}\right) )</td>
<td>( \sigma_E = 2\left(\lambda - \lambda^{-3}\right)\left(C_{10} + C_{01}\right) )</td>
</tr>
<tr>
<td>Ogden</td>
<td>( \sigma_E = 2\sum_{i=1}^{n} \mu_i \lambda^{(a_i)}(1 - \lambda^{-1.5a_i}) )</td>
<td>( \sigma_E = 2\sum_{i=1}^{n} \mu_i \lambda^{(a_i)}(1 - \lambda^{-1.5a_i}) )</td>
<td>( \sigma_E = 2\sum_{i=1}^{n} \mu_i \lambda^{(a_i)}(1 - \lambda^{-1.5a_i}) )</td>
</tr>
<tr>
<td>Neo-Hooke</td>
<td>( \sigma_E = 2C_{10}\left(\lambda - \lambda^{-3}\right) )</td>
<td>( \sigma_E = 2C_{10}\left(\lambda - \lambda^{-3}\right) )</td>
<td>( \sigma_E = 2C_{10}\left(\lambda - \lambda^{-3}\right) )</td>
</tr>
</tbody>
</table>

Table 5. The parameters list of the Ogden model obtained by the three fitting methods in ABAQUS

<table>
<thead>
<tr>
<th>Model</th>
<th>Group</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
<th>( \mu_4 )</th>
<th>( \mu_5 )</th>
<th>( \mu_6 )</th>
<th>( \mu_7 )</th>
<th>( \mu_8 )</th>
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<tr>
<td></td>
<td>Para._B</td>
<td>5.5072</td>
<td>-4.2385</td>
<td>0.00000318</td>
<td>3.5960</td>
<td>3.8636</td>
<td>-16.1179</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Para._C</td>
<td>7.6405</td>
<td>0.0000406</td>
<td>-6.3775</td>
<td>0.8641</td>
<td>22.2048</td>
<td>0.6530</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2 Model Parameter Fitting

The parameters of the constitutive model are fitted by three fit methods. Method A only uses UT test data to fit parameters. Method B uses both UT and BT test data to fit parameters. Method C uses UT, BT and PT test data to fit parameters. The abbreviation involved in later section as shown in Table 3.

Each model’s parameters are obtained from the test data by the above three fitting methods. The parameters are shown in Table 4 and Table 5.

Then, the tensile stress-strain curves are fitted by the parameters of material model obtained by the three fitting meth-
ods, and compared with the experimental curves. The engineering stress-strain curves of Yeoh model for uniaxial tension, biaxial tension and planar tension are fitted combining the equations $\sigma_E = \sigma_E(\lambda)$ in Table 2 and the parameters in Table 4 and Table 5, as shown in Fig. 2.

4. Fitting Accuracy Analysis

By utilizing the goodness of fit between the fitting curve and the experimental curve, the ability of each model to describe the mechanical behavior of material using parameters obtained from the three different fitting methods is evaluated and analyzed. The goodness of fit between the fitting curve and experimental curve in different strain range interval is estimated for Mooney-Rivlin model, Ogden model, Neo-Hooke model, Yeoh model and Arruda-Boyce model (the goodness of fit is characterized by the coefficient $R^2$, when the coefficient $R^2$ is 1, it means that the fitting curve and the experimental curve are completely coincident, higher fit goodness and better model fitting).

For a set of data $(x_i, y_i)$ and the corresponding model predicted value, $\hat{x}(x_i, \hat{y}_i)$ the calculation formula of the coefficient of determination, $R^2$ is defined as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} y_i^2}$$

(20)

4.1 Determination of Optimal Parameter Fitting Method

The goodness of fit of the material model between the fitting curve and experimental curve, using parameters obtained by the three fitting ways, is analyzed in different strain range interval to determine the optimal parameter fitting method for each model. The strain interval is divided by 10%, as shown Table 6.

The trend curves of $R^2$ in different strain interval are shown in Fig. 3, and the full name of the abbreviation is shown in Table 7.

It should be noted that in Fig. 3, A_BT uses the right vertical axis. A_BT has a very low value of $R^2$ and poor goodness of fit, which indicates that the Yeoh model with parameters fitted by method A cannot express the biaxial tension accurately. It is not found that the fitting accuracy of method C has significant improvement comparing with method B since the curves of method C and B are almost the same. Therefore, for Yeoh model, the best choice for parameters fitting is method B.

From the point of view of strain energy potential equation, Yeoh model is only high order related to $I_1$, the fitting effect of the model is affected by the parameter fitting method. Therefore, the uniaxial tension and biaxial tension data need to be combined to fit parameters of Yeoh model, which can make the model fit both the accuracy and economy.
Table 6. Interval divided

<table>
<thead>
<tr>
<th>Divided namely</th>
<th>Strain range</th>
<th>Divided namely</th>
<th>Strain range</th>
</tr>
</thead>
<tbody>
<tr>
<td>interval 1</td>
<td>0%~10%</td>
<td>interval 5</td>
<td>40%~50%</td>
</tr>
<tr>
<td>interval 2</td>
<td>10%~20%</td>
<td>interval 6</td>
<td>50%~60%</td>
</tr>
<tr>
<td>interval 3</td>
<td>20%~30%</td>
<td>interval 7</td>
<td>60%~70%</td>
</tr>
<tr>
<td>interval 4</td>
<td>30%~40%</td>
<td>interval 8</td>
<td>70%~80%</td>
</tr>
</tbody>
</table>

Table 7. Full name of the abbreviation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_UT/B_UT/C_UT</td>
<td>The R² value of goodness of fit in different strain interval between Test_UT and A_calculate_UT/B_calculate_UT/C_calculate_UT</td>
</tr>
<tr>
<td>A_BT/B_BT/C_BT</td>
<td>The R² value of goodness of fit in different strain interval between Test_BT and A_calculate_BT/B_calculate_BT/C_calculate_BT</td>
</tr>
<tr>
<td>A_PT/B_PT/C_PT</td>
<td>The R² value of goodness of fit in different strain interval between Test_PT and A_calculate_PT/B_calculate_PT/C_calculate_PT</td>
</tr>
</tbody>
</table>

Figure 3. The goodness of fit between fitting curves and experimental curves of Yeoh model in different strain range interval

4.2 Model Accuracy and Optimum Strain Range

With the optimal parameter fitting, the goodness of fit of each model was analyzed in different strain range (coefficient of determination R² method) to investigate the model accuracy in different strain range.

From the foregoing, it is known that the best approaches to fit parameters of the five models including Mooney-Rivlin, Ogden, Neo-Hooke, Yeoh and Arruda-Boyce are successively method B, method B, method A, method B and method A. According to the tensile type, the R² values of goodness of fit for each model with parameters, obtained by the optimal fitting method, are contrasted and analyzed successively, as shown in Fig. 4.

Fig. 4(a) shows that the R² value of the goodness of fit of the five models for fitting uniaxial tension are basically the same, and the fitting precision are poor within 10% strain, the fitting accuracy of Yeoh model is slightly better in low strain, but a bit weak in the large strain interval more than 60%. Fig. 4 (b) shows the goodness of fit of the five models for fitting biaxial tensile, fitting accuracy of Yeoh model is better above strain of 20%, fitting accuracy of Ogden model is better in the 10%~60%, and Yeoh model, Neo-Hooke model and Arruda-Boyce model performs slightly better within 10% strain. The goodness of fit of planar tension is shown in Fig. 4(c), Yeoh model is the best within strain 10%, Ogden model is better in 10%~40% strain range, while the fitting precision of Yeoh model is also good in more than 30% strain.

As a conclusion, shown in Fig. 5, Yeoh model has a better fitting accuracy of uniaxial tension, biaxial tension and planar tension in 60% strain, and it has better fitting precision in the five common used hyperelastic constitutive models in low
strain range. Ogden model’s fitting precision is better in 10%~40% strain range but the worst within 10% strain. The rest of the model’s fitting precision are slightly inferior to the prior two models, Moone-Rivlin model fits slightly well only in 10%~20% strain range, Neo-Hooke and Arruda-Boyce model fit well within 10% strain, and the goodness of fit of this two models are basically the same, but Arruda-Boyce model is very complex, not recommended.

5. Simulation and Verification

In the application of tire analysis, the rubber materials of different parts of tire have different deformation under inflation, small deformation and medium deformation, also large deformation conditions. Combined with the compared results of the model accuracy in the last chapter and the actual situation, this paper uses the Yeoh model as the material model of tire analysis. To determine the model accuracy of tire simulation analysis, this paper uses the Yeoh model as the material model of the model accuracy of tire simulation analysis. The simulation results are shown in Table 9.

The stiffness values were calculated by the model with parameters fitted by the three methods are not the same. Compared to the simulation results with Para._A, the simulation stiffness values with Para._B are closer to the test stiffness values, and the simulation precision is improved. But compared to the simulation results with Para._B, the simulation stiffness values with Para._C change a little.

The analysis of relative standard deviation of the simulation stiffness value, were calculated with the three sets of parameters, based on the test stiffness values are presented in Fig. 7. It is obviously observed that the relative errors with Para._A are all larger, and the relative errors with Para._B are significantly reduced compared to the relative errors with Para._A. In addition, the relative errors with Para._C have been decreased but not obvious. It can improve the simulation precision of the FEM stiffness model both with the Para._B and Para._C, but taking into account the cost of obtaining the parameters; using the Para._B to carry out the simulation can both improve the accuracy of the simulation model and save the test cost. Therefore, it is high precision and economy to simulate with Para._B, and this is confirmed with the conclusion that Yeoh model uses method B as the optimal parameter fitting method in last chapter.

![Simulation and Verification](image)

Figure 4. Contrast of the goodness of fit for each model
Figure 5. Optimum strain range of each model

Table 8. Parameters of Yeoh model used in simulation

<table>
<thead>
<tr>
<th>Group</th>
<th>$C_{A1}$</th>
<th>$C_{A2}$</th>
<th>$C_{A3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Para. A</td>
<td>0.8505</td>
<td>-0.4531</td>
<td>0.2372</td>
</tr>
<tr>
<td>Para. B</td>
<td>0.7246</td>
<td>-0.1786</td>
<td>0.0664</td>
</tr>
<tr>
<td>Para. C</td>
<td>0.7132</td>
<td>-0.1719</td>
<td>0.0685</td>
</tr>
</tbody>
</table>

Figure 6. Finite element model of tire

Figure 7. Error of simulation value based on test value
6. Conclusions

The fitting effect of the five material model of hyperelastic rubber with different parameter fitting method are contrasted and analyzed in different strain range and the conclusion as follows.

a. The method of parameter fitting has influence on the fitting effect of model, and mainly on the biaxial tension. It mainly depends on the relationship between the SEP function and I1 that whether the model is affected by the parameters fitting method, and the conclusion is as follows:

1) The SEP function of Neo-Hooke model is only first order related to I1, so in the application of the model, only the uniaxial tension data is used to fitting the parameters of model.

2) The SEP function of Mooney-Rivlin model is both first order related to I1 and I2. The SEP function of Ogden model is high order related to both I1 and I2. The SEP function of Yeoh model is high order related to I1. Therefore, the fitting effect of these three models is affected by the parameter fitting method. In the application of these three models, it is necessary to use the data of both uniaxial tension and biaxial tension to fit the model parameters.

3) The SEP function of Arruda-Boyce model is high order related to I1, but due to the coefficient of high order term of I1 is very small and far less than I1, weaken the influence of high order term of I1 on model effect, so the influence of parameter fitting method on this model is not obvious. When this model is applied, the parameters can be fitted only by uniaxial tension data.

b. When the parameters of each model are fitted by the best fitting method, Yeoh model has a better fitting accuracy of uniaxial tension, biaxial tension and planar tension in 60% strain, but it has better fitting precision in the five common used hyperelastic constitutive models in low strain range. Ogden model has better fitting accuracy in 10%~40% strain range, but the worst effect in 10% strain range.

The rest of the five model’s fitting precision are slightly inferior to the prior two models. Moone-Rivlin model fits slightly well only in 10%~20% strain range. Neo-Hooke and Arruda-Boyce model fit well in 10% strain range, and the goodness of fit of these two models are basically the same, but Arruda-Boyce model is very complex, not recommended.

c. Yeoh model is used to simulation and analysis of FEM model with the parameters fitted by the three kinds of fitting methods, and compared with the experimental results. It confirms that this model has the best precision and economy to do stiffness simulation with parameters fitted by method B, and it is corroborated with the conclusion a that Yeoh model uses method B as the optimal parameter fitting method, that is to say that fitting parameters of Yeoh model with both UT and BT test data is the best method..

References

[17] Kim RY, Crasto AS. An Improved Test Specimen to De-


